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Results are reported for an experimental investigation of the transition flow region in a free axisymmetric jet of incompressible viscous fluid ejected from a profiled nozzle and long cylindrical tube.

Not enough attention has been devoted to flow in the laminar-turbulent transition region in free jets of an incompressible viscous fluid. This is indicated, in particular, by the lack of detailed data in monographs and surveys concerned with the appearance of turbulence [1-3] and with free jets [4-6].

Isolated interesting discussions of the transition in jet flow occur in several articles, where the phenomenon is discussed from various viewpoints.

In [7-9], the jet stability problem has been solved within the framework of the linear theory. Observations of the transition and measurements of the velocity (mean and pulsation) have been discussed in [10-20]. In certain cases, attempts have been made to determine the characteristic parameters, the length of the transition section, etc. Qualitative aspects of the relationship between the effective viscosity and the Reynolds number during the transition have been discussed in [20].

Certain authors, in addition to studying the transition in jets in "natural" form have reported results obtained by applying acoustical oscillations to the transition. In these studies (see, in particular, the photographs of [21]) certain resonance effects have been demonstrated, and detailed spectral data given [12].

On the whole, however, the investigation of the laminar-turbulent transition in free jets is far from completed, particularly from the quantitative viewpoint. The data given here represent a certain contribution in this area.

We note that our problem is of considerable theoretical interest, since we are concerned with a relatively little-studied transition region for a free flow far from solid walls.

The problem is of practical interest for high-temperature plasma jets (fairly low Reynolds numbers), for problems of fluidics, and for flame combustion. Moreover, interaction effects of molecular and molar exchange appear on the initial sections of free turbulent jets (particularly when the initial turbulence is artificially suppressed). Thus viscosity may have a significant influence for Reynolds-number values (computed from outlet parameters) of the order of 10^3 - $2 \cdot 10^4$. This is important for proper interpretation of experimental results and, in particular, generalization of the effective transport coefficients obtained. The common notion that motion in free jets is turbulent almost from the very beginning (from the nozzle edge) for this region of Re values cannot be assumed valid for the general case. As an illustration, in Fig. 1 we have shown an air jet (carrying smoke) leaving a tube ($l/d = 100$) for three values of Re. The photograph clearly shows initial sections in which the flow is nearly laminar. Visible turbulence appears some distance from the outlet. As we shall show, these and similar visual observations are supported by direct measurement of average and pulsation quantities.

The nature of the flow and the developmental dynamics of the jet are substantially influenced by the initial conditions: the velocity profile at the nozzle outlet, the turbulence level, etc. In this connection, it is of interest to perform an experiment with various initial velocity profiles, and with a regulated initial turbulence level. Below we shall give data obtained for two limiting cases of a stabilized (for a given value

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TABLE 1. Parameters of Investigated Regimes

Characteristic		Range of parameter variation			
type of head	outlet diameter, mm	max. outlet velocity, u_{m0} , m/sec	mean outlet velocity, u_{av} , m/sec	Reynolds number $Re_0 = \frac{u_{av} a}{\nu}$	Strouhal number $Sh = n d / u_{av}$
Tube, $l/d = 100$	3,8	10—53	6—42	$1,6 \cdot 10^3 - 10 \cdot 10^3$	—
The same	6	13,5—38	10—30	$4 \cdot 10^3 - 11,7 \cdot 10^3$	—
"	10	16—42	12,5—32,5	$8,5 \cdot 10^3 - 22 \cdot 10^3$	—
"	14	20—52	15,5—38	$14 \cdot 10^3 - 33 \cdot 10^3$	—
Nozzle (compressed along cross section)	3,8	8—68	8—68	$12 \cdot 10^3 - 17 \cdot 10^3$	0—0,12

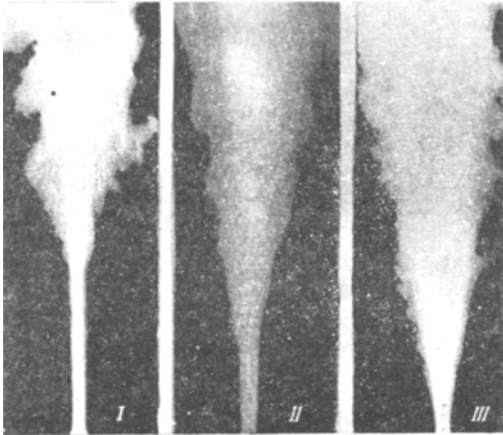


Fig. 1. Development of free jets for various Reynolds numbers (flow from a tube): I) $Re = 2000$; II) 5000; III) 8000.

of Re) velocity profile with flow from a long tube ($l/d = 100$) and a nearly uniform profile for flow from a profiled nozzle. Figure 2a shows the initial velocity profiles for the different flow conditions.

For nozzle flow, in addition to jets with "natural" turbulence, jets with increased initial turbulence levels have been investigated; here an effective turbulence generator in the form of a mechanical pulsator was used [22, 23]. Apparently, no such experiment with active mechanical action on the transition conditions had been performed previously.

To provide a general characterization of the experiments, Table 1 gives the basic parameters of the setups and the regimes investigated.

In all cases, the measurements were taken in a stream of room-temperature air flowing into an unbounded space filled with air at the same temperature (and atmospheric pressure).

The experiments with a profiled nozzle were carried out both in the absence of a turbulence generator ($Sh = 0$), and for various speeds of the turbulizing disc ($n = 0 - 10 \cdot 10^3$). A turbulence generator resembling the one described previously [22] was installed in a wide section of the line ($d_l = 32$ mm), 90 mm away from the nozzle edge (the length $l_n = 32$ mm). The diameter of the turbulizing disc was $d_t = 28$ mm.

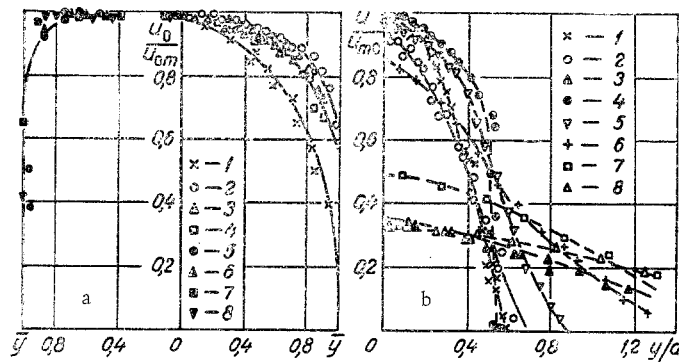


Fig. 2. Initial velocity profiles for flow from tube [1-4] and nozzle (1-8) (a), and velocity distribution in cross sections (flow from tube) (b): a) 1) $Re_0 = 2700$; 2) 4750; 3) 18,300; 4) 35,600; 5) 2000; 6) 3000; 7) 4600; 8) 26,000; b) for $Re_0 = 2070$: 1) $x/d = 0$; 2) 13; 3) 19; for $Re_0 = 23,200$: 4) $x/d = 0$; 5) 2; 6) 7; 7) 13; 8) 19.

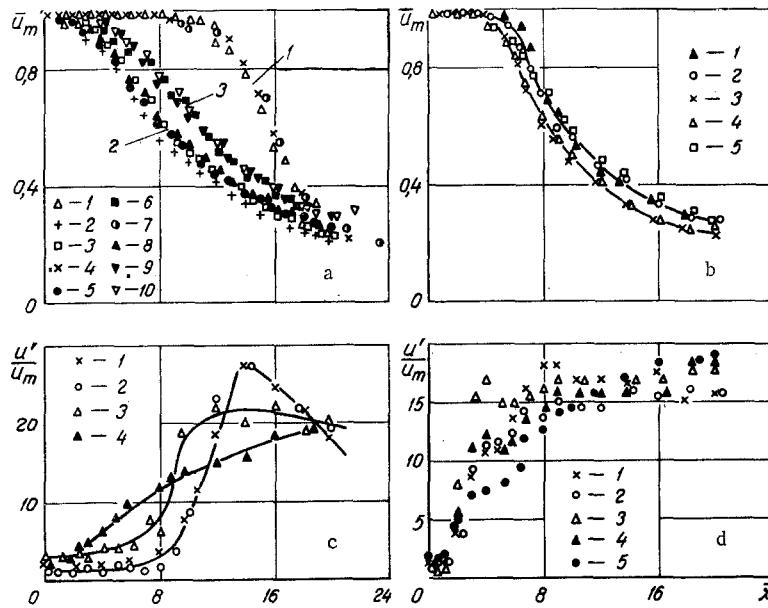


Fig. 3. Variation in velocity and turbulence intensity along jet axis for various values of Re : a) flow from tube: 1) $Re_0 = 2070$; 2) 4000; 3) 25,850; 4) 2700; 5) 4650; 6) 18,300; 7) 3000; 8) 5500; 9) 21,600; 10) 35,600; b) flow from nozzle: 1) $Re_0 = 2000$; 2) 3000; 3) 4600; 4) 6290; 5) 13,200; c) flow from tube: 1) $Re_0 = 1770$; 2) 2900; 3) 4250; 4) 8300; d) flow from nozzle: 1) $Re_0 = 2000$; 2) 3000; 3) 4600; 4) 6200; 5) 20,600.

The dynamic pressure was measured by ordinary Pitot tubes with 0.38 mm inside diameter and 0.12 mm wall thickness. The pulsations in the longitudinal component of the velocity vector, $u' = \sqrt{u'^2}$, were measured by means of a type ÉTAM-3A hot-wire anemometer, with 15 μ diameter wire.

In all cases, measurement accuracy was checked by seeing that the momentum density flow remained constant, $I_x = 2\pi \int \rho u^2 y dy$; I_x was maintained to within about 3-7% (owing to the difficulty of the measurements, the maximum deviation refers to the smallest value $Re \approx 1.5 \cdot 10^3$ in the experiment).

Figure 3 shows the loss in average flow velocity $\bar{u}_m = u_m / u_{m0}$ on the jet axis for flow from a tube (Fig. 3a) and from a nozzle for $Sh = 0$ (Fig. 3b), with various values of Reynolds number.

The first thing we notice is the nonmonotonic relationship between the intensity of jet attenuation and Re . As we see from Fig. 3a, for example, beginning at $Re = 3 \cdot 10^3$ (curve 1), an increase in Re results initially in a sharp increase in attenuation (transition to curve 2 for $Re = (4-6) \cdot 10^3$) and then to a reduction in attenuation (curve 3 for $Re = 15 \cdot 10^3$). In these experiments, each value of Re naturally corresponded to its own initial velocity profile (Fig. 2a); the change in the profile (its "filling") took place monotonically, from a Poiseuille parabola ($Re = 2 \cdot 10^3$) to a characteristic turbulence profile of the one-seventh law type.

As we see from Fig. 3b, a similar nonmonotonic relationship between jet attenuation and the value of Re is also found for flow from a nozzle where the initial velocity profile remains nearly constant. The curves for different Re values differ less in this case than for the tube, however.

Figure 2b clearly shows the rearrangement of the flow when Re varies within the indicated range of values; it shows the distribution of the mean velocity $\bar{u} = u / u_m$ in jet cross sections (depending on $\bar{y} = y/d$). For $Re = 2070$, up to the section $\bar{x} = 13$, the velocity profile is nearly parabolic; at $\bar{x} \geq 19$, it acquires the form characteristic of free turbulent flow. At higher values ($Re = 23,200$) the velocity profile changes at $\bar{x} = 2$, and as we move away from the orifice, it corresponds to turbulent stream flow.

A more complete picture of the change in average velocity on the jet axis emerges in Fig. 4a, which shows the relationship $\bar{u}_m = f(Re)$ for various values of \bar{x} . It is illustrated schematically in the upper right-hand corner.

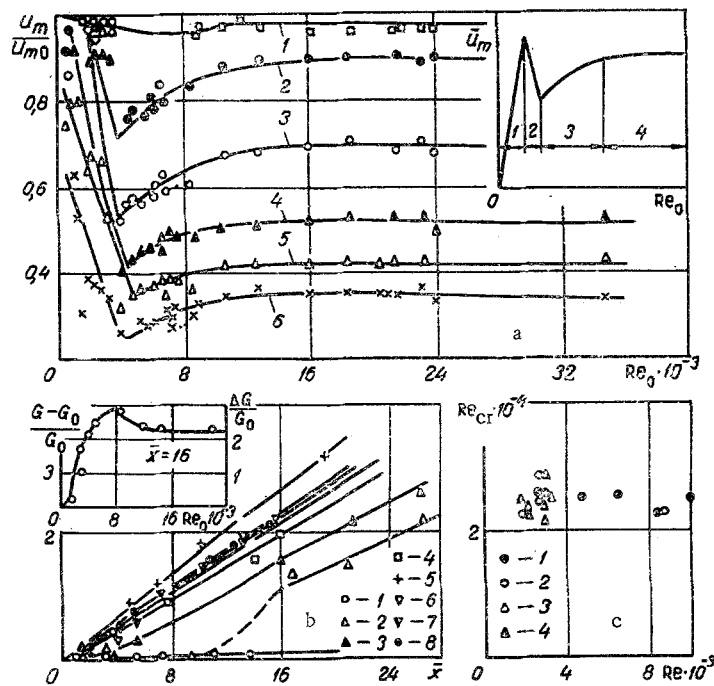


Fig. 4. Relationship $\bar{u}_m = f(Re)$ for jet leaving tube (a), variation in associated mass along jet axis (for tube) (b), and $Re_{x cr} = f(Re_0)$ (c): a) 1) $\bar{x} = 3$; 2) 6; 3) 9; 4) 12; 5) 15; 6) 18; b) 1) $Re_0 = 2070$; 2) 3000; 3) 3440; 4) 4750; 5) 8400; 6) 11,700; 7) 14,000; 8) 23,200; c) 1) based on results of measurements of turbulence intensity; 2) based on variation in relative half-width of jet; 3) from data of [14]; 4) from data of [24].

The data obtained require a fairly detailed description.

As we see from Fig. 4a, for any specific distance \bar{x} , we can arbitrarily isolate four characteristic segments on the $\bar{u}_m = f(Re)$ curve.

In accordance with the analytical solution to the problem of laminar flow [6], on the first segment the velocity rises in proportion to Re . This results from the relatively smaller role played by the viscosity (the increased role of inertia) in the jet, which increases the effective range.

On the second segment, owing to loss of stability and the appearance of intense pulsations, we find a qualitative change in the nature of the relationship: \bar{u}_m rises sharply with increasing Reynolds number. This critical transition is characteristic for flow in a pipe, for example.

On the third segment, which also pertains to the transition region (but which is considerably longer in terms of Re) we find an increase in \bar{u}_m with increasing Re . In other words, in this flow region, as with laminar motion, the intensity of stream attenuation with increasing Reynolds number again drops. From the physical viewpoint, there is, as it were, a stabilization of the effective turbulent viscosity.

Finally, the fourth segment corresponds to a region of developed turbulent flow in the free jet. It is characterized by self-similarity for Re ; the value of u_m remains constant (for a given x/d).

A similar, although smoother, pattern is found for flow from a profiled nozzle.

In all cases, we have an approach "from below" to self-similar turbulent flow. This situation, an increase in attenuation with decreasing Re (on the third segment), has also been noted elsewhere [15]. The various data are compared in Fig. 5c.

It must be emphasized that the transition pattern for the jet (Fig. 4a) has a feature that distinguishes it from other types of flow. When the laminar motion loses stability, the intensity of exchange in the transition region increases so much that it exceeds the value corresponding to developed turbulent motion. This is also reflected in the integral characteristics of transition.

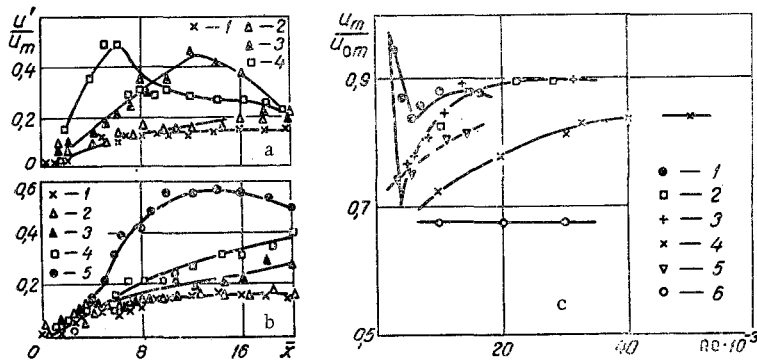


Fig. 5. Variation in turbulence intensity in a jet for various Strouhal numbers (a, b), and comparison of experimental data for dependence of velocity at jet axis ($x/d = 6$) on Reynolds number (c): a) for $Re_0 = 3000$: 1) $Sh = 0$; 2) 0.11; 3) 0.029; 4) 0.043; b) for $Re_0 = 6250$: 1) $Sh = 0$. 2) 0.0076; 3) 0.0138; 4) 0.020; 5) 0.0275; c) for $Sh = 0$: 1) nozzle, 3.8 mm diameter; 2) data of [15]; 3) tube, 3.8 mm diameter; 4) data of Kelmanson (nozzle); for $Sh = 0.012$: 5) nozzle, 3.8 mm diameter; for $Sh = 0.1$: 6) data of Kelmanson (nozzle).

Figure 4b gives the associated mass $\Delta G = G - G_0$ (as a fraction of the initial flow rate G_0) as a function of \bar{x} and Re ($G_0 = 2\pi \int_0^{r_0} \rho u_0 y dy$ and $G = 2\pi \int_0^{\infty} \rho u y dy$ are the integral flow rates in the initial cross section and arbitrary section). The effect is also pronounced here.

To provide a more complete description of the transition, we shall give some results of hot-wire anemometer measurements of the pulsations in the longitudinal velocity component along the depth axis. Figure 3c shows such data for flow from a tube with various values of Re . These data are in qualitative agreement with those given above. The lower Re , the earlier the intense increase in pulsation begins, and the greater the maximum turbulence intensity (which exceeds the value for developed turbulence at $Re = 8 \cdot 10^3$). Data that are similar in nature but display significantly more spread are shown in Fig. 3d for flow from a nozzle. As for the mean velocity, the influence of Re is less noticeable here.

Since the mixing zone is so small, it was not possible to measure u' in the jet cross sections, which would have given a more complete picture. Even so, there is no doubt that the results of the measurements of the average and pulsation characteristics are in agreement.

To provide a quantitative characterization, it is desirable to introduce the critical Reynolds number, as for other transition cases. Processing of the experiments on flow from a tube shows that it is best to take the number Re_x for this value; it is calculated from the local velocity on the jet axis and the distance from the head to the flow zone within which a vigorous change in the characteristics occurs. Despite the rather arbitrary nature of such a definition, as we see from Fig. 4c the values of $Re_{x_{cr}}$ found from the various characteristics, fall within a fairly narrow range of values, $Re_{x_{cr}} = (20-25) \cdot 10^3$. Thus for a tube, $Re_{x_{cr}} \approx \text{const}$ for a fairly wide range of variation in Re_0 , computed from the exit parameters.

The $Re_{x_{cr}} \approx \text{const}$ relationship is easily converted to an expression for the length of the laminar-flow segment in the jet (before the transition),

$$\left(\frac{x}{d}\right)_{cr} \approx (20-25) \cdot 10^3 \cdot \frac{u_m}{u_{m_0}} \frac{1}{Re_0}.$$

As for flow from a nozzle, it is difficult to obtain an adequate definition of $Re_{x_{cr}}$ (see Fig. 3b, d).

Additional experiments were carried out with forced oscillations superposed on the jet by means of a mechanical turbulence generator; the results showed that for small $Re = 3 \cdot 10^3$, Sh has the same strong influence as for developed turbulent motion [22]. In other words, the addition of forced pulsations results in earlier loss of flow stability and severe attenuation.

Measurements of velocity pulsations on the jet axis (Fig. 5) show that as Sh increases, the maximum value of u' rises, while the location of the pulsation peak shifts toward the nozzle. This effect is more pronounced at lower Re values.

From the qualitative viewpoint, the mechanical turbulence generator (as in the case of acoustical oscillations [21]) results in earlier onsets of the transition.

To conclude, let us look at some results (Fig. 5c) obtained by us and by other authors for the relationship between the velocity on the jet axis and the Reynolds number. As we see, in all cases of flow from a tube or nozzle, in addition to developed turbulent flow at $Sh = 0.1$, there is an increase in the velocity on the axis in a given cross section when Re increases. This region corresponds to the third segment of the curve of Fig. 4a. The sections of critical drop in the value of u_m/u_{m0} are also shown in the region of initial Re values for flow from a tube.

Thus the initial conditions (nozzle shape, velocity profile, pulsation intensity, etc.) have a substantial influence on the flow in a jet within the region next to the nozzle ($x/d \leq 10-12$, roughly). This must be taken into account when we analyze data pertaining to this flow region for $Re \leq (20-30) \cdot 10^3$.

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